

On the perseverance of a quasi-two-dimensional eddy-structure in a turbulent mixing layer †

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Strong external disturbances were introduced into a mixing layer in order to test the formation of the quasi two-dimensional coherent eddies and their survival under less than ideal conditions. Velocity and temperature correlation measurements, flow visualization, and the simultaneous use of a large number of sensors suggest that these eddies are very stable in the range of Reynolds numbers considered and they persevere in spite of the external buffeting imposed. Some measurements were carried out in a mixing layer between two parallel streams and some in a mixing layer entraining quiescent surrounding fluid. In both cases the large eddies could be described as vortex rolls spanning the test section; these rolls may be contorted and sometimes skewed, but they are basically two-dimensional.

1. Introduction

The coherent large-eddy structure occurring in a two-dimensional, turbulent mixing layer at moderate Reynolds numbers appears to be two-dimensional. Shadow-graph pictures taken by Brown & Roshko (1974) show an array of large vortices which span the entire mixing region. Correlation measurements taken just outside the turbulent region by Oster, Wygnanski & Fiedler (1977) indicate also that the large eddy structure is indeed quasi two-dimensional. The existence of the two-dimensional structure not only facilitates the understanding of the entrainment process but also indicates the way by which a mixing process can be artificially enhanced. However, the above-mentioned experiments were carried out in an environment in which the ambient turbulence level was low and the free stream velocity was uniform. These conditions seldom exist in practice so it became important to establish whether the two-dimensional character of the large eddies can be maintained under less favourable conditions. Chandrsuda *et al.* (1978) suggest that the two-dimensional character of these eddies is a relic of the transition process and that the flow becomes highly three-dimensional whenever the free-stream turbulence is high or the boundary layer on the splitter plate becomes turbulent. Pui & Gartshore (1979) propose that the periodicity of the largest eddies may result from external forcing like the vibration of the splitter plate, but the structure of the large eddies is not even moderately two-dimensional.

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We thus set out to test the sensitivity of the large eddy structure to external conditions. There is no attempt to map the details of the flow, but simply to establish the degree of two-dimensionality for various initial conditions by observing the flow, and by measuring the correlation coefficients $R(0, 0, \Delta z, \tau)$, where Δz is the spanwise distance between the measuring stations and τ is the time delay.

Most of the measurements were taken in a wind tunnel facility described by Oster *et al.* (1977). The flow is considered to be incompressible because the velocity of the faster stream, U_2 , is 15 m/s and the velocity ratio between the streams is maintained at $U_1/U_2 = 0.4$. The Reynolds number based on downstream distance is of the order 10^6 . Observations in a single stream mixing layer, $U_1/U_2 = 0$, were carried out in a facility described by Fiedler (1975) at a free stream velocity of $U_2 = 10$ m/s and $Re_x \leq 0.5 \times 10^6$. In the unforced case this mixing layer was fully turbulent and self-preserving for $Re_x > 0.2 \times 10^6$.

2. Some details about the experiment

The trailing edge of the splitter plate in the two-stream mixing layer is milled out of aluminium at an included angle of 3° . The boundary layer on both sides of the splitter plate is normally laminar although on the high-velocity side occasional turbulent spots were observed. The turbulence of the free stream is approximately 0.2% and the power spectra measured in both free streams contain peaks at the following frequencies: (i) 11 Hz, being the organ pipe frequency (see also Fiedler *et al.* 1977), (ii) 223 Hz (in the high-velocity stream), (iii) 30 Hz (in the low-velocity stream), these frequencies correspond to the blade-passing frequency of the appropriate fan.

The vibrations of the splitter plate were monitored with an accelerometer (Bruel & Kjaer, Model 2626), the predominant frequencies at which the splitter plate vibrates are 112, 56, 84 (the spectral peak at 84 is rather broad). Presumably 56 is the lowest frequency of vibration, 112 should be the first harmonic frequency, and the intermediate frequency of 84 results from $\frac{2}{3}f_1$. Stiffening the splitter plate with a brass plate (on which vortex generators were machined) eliminated the intermediate spectral peak entirely.

Four different configurations affecting the initial conditions were studied (figure 1).

(i) This configuration was studied for two cases: (a) a trip wire 1.6 mm in diameter was placed 11.2 mm upstream of the trailing edge ($x/d = -7$) on the high-velocity side of the splitter plate; (b) the same trip was moved to $x/d = -30$.

(ii) A large row of vortex generators was fastened to the splitter plate 500 mm upstream of the trailing edge, in order to change the character of the boundary layer on the high-velocity side.

(iii) A brass ribbon stretched between two voice coils of large woofers was placed downstream of the trailing edge spanning the entire test section. The voice coils moved up and down with a phase difference of 180° oscillating the ribbon in a skewed fashion, in relation to the trailing edge of the splitter plate. The oscillations of the ribbon were tested *in situ* using stroboscopic light. It was observed that the ribbon oscillated in its primary mode and the angle it formed with the trailing edge was found to be $\alpha = 0.2; 0.4; 0.8^\circ$ for the three amplitudes considered. The frequency of the oscillation was 40 Hz.

(iv) A large biplanar grid made of steel rods 3.2 mm in diameter and having a

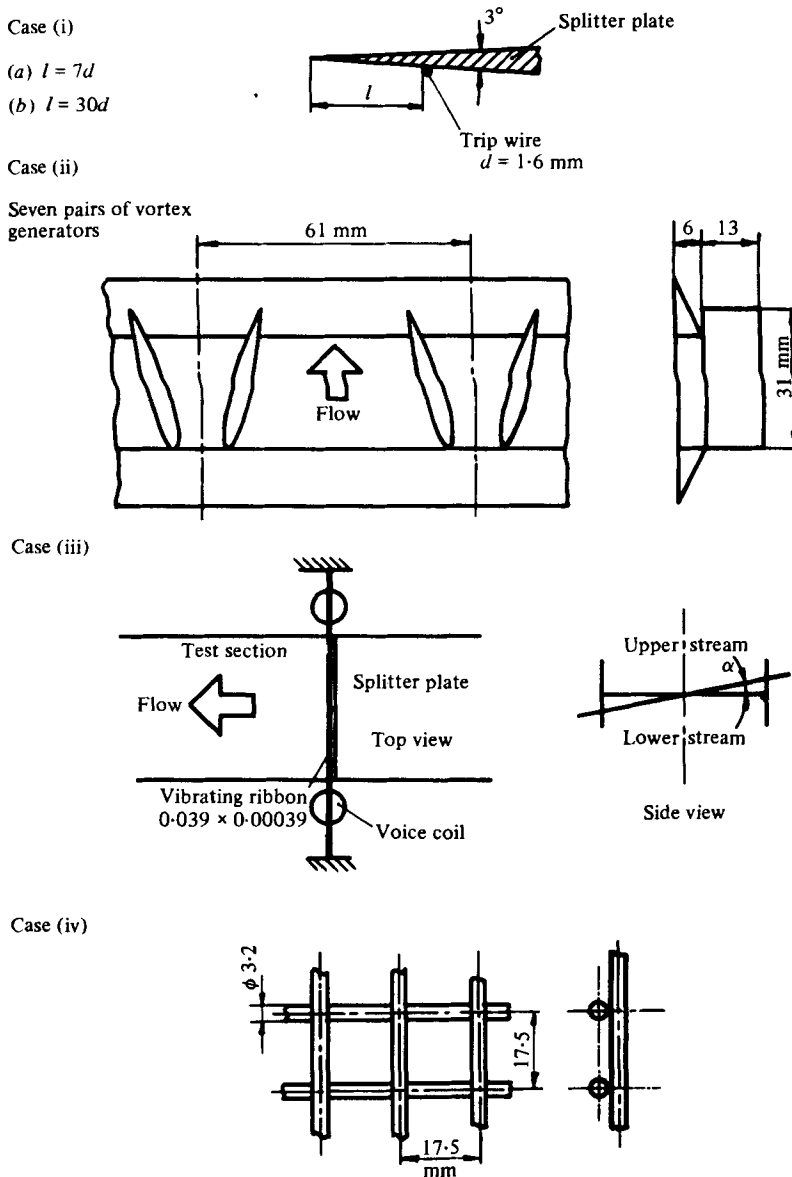


FIGURE 1. The geometric configurations of the cases considered.

solidity ratio of 0.34 was placed 250 mm upstream of the trailing edge in both streams. The measured level of turbulence in both streams at $x = 0$ was more than 3%.

The single-stream mixing layer originated at a backward-facing step like the one used by Wygnanski & Fiedler (1970). The entrained flow came from a large room after passing through a screen cage which was large in comparison with the test section. The mixing layer did not reattach to any solid surface. The flow leaving the step was turbulent because the boundary layer was tripped by a wire (1.6 mm in diameter) and a strip of very coarse sandpaper glued to the surface. The free stream

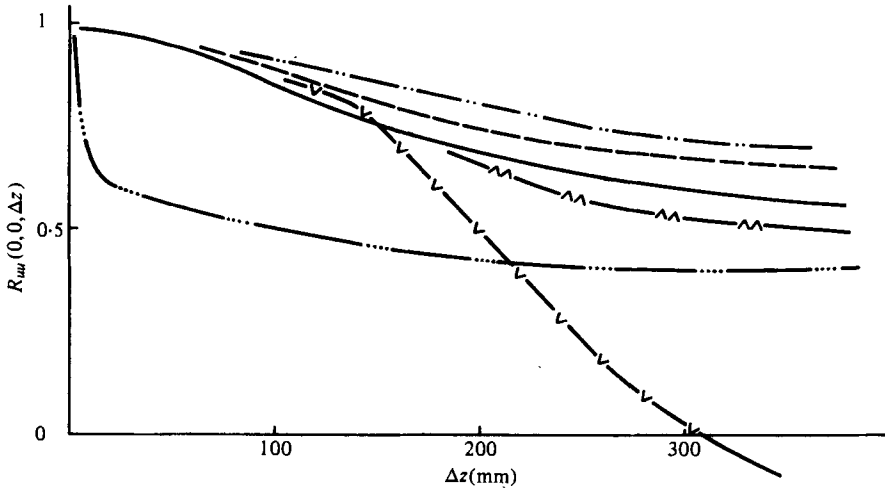


FIGURE 2. $R_{uu}(0, 0, \Delta z, 0)$ correlations measured mostly in the potential flow adjacent to the mixing layer $U_1/U_2 = 0.4$ and $x = 1100$ mm. —, case (i) (a), ---, case (i) (b), trip wire; - · - · -, case (ii); - Δ - Δ - case (iii) (b), angle 0.4° ; - ∇ - ∇ - case (iii) (c), angle 0.8° ; ·····, case (iv).

was heated to approximately 20°C above the ambient room temperature. The excess temperature was used as a passive contaminant which facilitated the assessment of the two-dimensionality of the large coherent structures.

3. Results

The perseverance of two-dimensionality of the large coherent structures in a mixing layer between two parallel streams was established by correlating the streamwise and transverse components of velocity. Initially, the coefficient $R_{uu}(0, 0, \Delta z, 0)$ was measured at the outer edges of the mixing layer at two downstream distances $x = 500$ mm and $x = 1100$ mm. The intermittency factor γ at the measuring stations was less than 5% except in case (iv), for which the free-stream turbulence was so high that the concept of intermittency lost its meaning. Mostly potential fluctuations induced by the large vortex-rolls passing near the measuring stations are correlated in this way, some of the results are plotted in figure 2, and the correlation coefficients for separation distance $\Delta z/(\text{span of the wind tunnel}) = 0.63^\dagger$ are shown in table 1.

These results are indicative that an orderly structure, as first observed by Brown & Roshko (1974) exists, and it is quasi two-dimensional up to $Re_x \sim 10^6$. The disturbances imposed on the flow did not affect the two-dimensional character of the large eddies, although they might have affected their rate of growth and amalgamation, as well as their detailed turbulent structure.

Correlation coefficients at $x = 1100$ mm are usually higher than at $x = 500$ mm. Only in case (iii) ($\alpha = 0.8^\circ$) the correlation diminished with x , suggesting that the anti-symmetric oscillations of the ribbon result in a three-dimensional interaction between

\dagger The dimensionless separation distance $\Delta z/b$, where b represents the width of the layer; [$b = |y_{0.1} - y_{0.95}| \approx (0.075 \pm 10\%) (x + x_0)$] and x_0 is the location of the hypothetical origin was not used in this case because the correlation lengths are comparable to the cross-sectional dimension of the wind tunnel and might have been limited by it.

Case	High velocity side		Low velocity side	
	$x = 500$ mm	$x = 1100$ mm	$x = 500$ mm	$x = 1100$ mm
(i) (a)	0.44	0.58	0.45	0.55
(i) (b)	0.44	0.63	0.51	0.65
(ii)	0.55	0.71	0.55	0.59
(iii) ($\alpha = 0.2$)	0.33	0.43	0.36	0.44
(iii) ($\alpha = 0.4$)	0.23	0.49	0.32	0.32
(iii) ($\alpha = 0.8$)	-0.15	-0.25	0.05	-0.05
(iv)	0.27	0.44	0.45	0.62

TABLE 1. $R_{uu}(0, 0, \Delta z, 0)$ (at $\Delta z/\text{span} = 0.63$, and in the potential region outside the mixing layer at y corresponding to $\gamma < 5\%$).

adjacent vortices. In this case the helical pairing process suggested by Chandrsuda *et al.* (1978) may indeed occur.

For case (i)(b) it was ascertained that the flow had the time to reattach downstream of the trip wire, forming a turbulent boundary layer. Power spectra measured in this boundary layer were 'typical' of turbulent flows without any recognizable peaks, and the velocity profile obeyed the universal laws. In spite of it all $R_{uu}(0, 0, \Delta z)$ correlations were higher for this case than for case (i)(a). In case (ii) the vortex generators changed the mean velocity distribution and the turbulent intensity, yet the correlation coefficient on the high-velocity side increased even more (see also Fiedler 1975). We thus concluded that the existence of a turbulent boundary layer at the trailing edge does not inhibit the formation of the coherent structures nor their two-dimensional character.

The predominant frequency of the coherent structures at $x > 500$ mm is 43 Hz with the 'organ-pipe' frequency of 11 Hz being present. These frequencies do not correspond to the frequency at which the plate vibrates. Furthermore, attaching the heavy brass strip with vortex generators to the splitter plate changed the mode of vibrations of the plate, indicating that these vibrations did not affect the two-dimensional nature of the flow as might have been inferred from Pui & Gartshore (1979).

The purpose of the third experiment was to find if a 'helical-pairing' mode seen by Chandrsuda could be artificially stimulated. The present results indicate that a three-dimensional interaction can be induced at sufficiently high amplitudes of the 'vibrating ribbon' and the resulting negative correlation becomes even more negative with increasing x . The space-time correlation $R_{uu}(0, 0, \Delta z, \tau)$ is periodic, suggesting considerable measure of coherence in the large eddy-structure.

The disparity between the conclusions of Chandrsuda *et al.* (1978) and Pui & Gartshore (1977) and the present findings are too large to be attributed to an experimental error, and are too important to be ignored. For this purpose the correlation $R_{uu}(0, 0, \Delta z)$ in case (i)(b) was measured at two additional locations corresponding to local velocity $U = U_1 + 0.1(U_2 - U_1)$; $U = U_1 + 0.5(U_2 - U_1)$, where the intermittency factor is approximately 50 and 96% respectively. The correlations decreased rapidly with increasing γ (figure 3) and are in good agreement with measurements of other authors (e.g. Jones, Planchon & Hammersley 1973). The relatively small integral

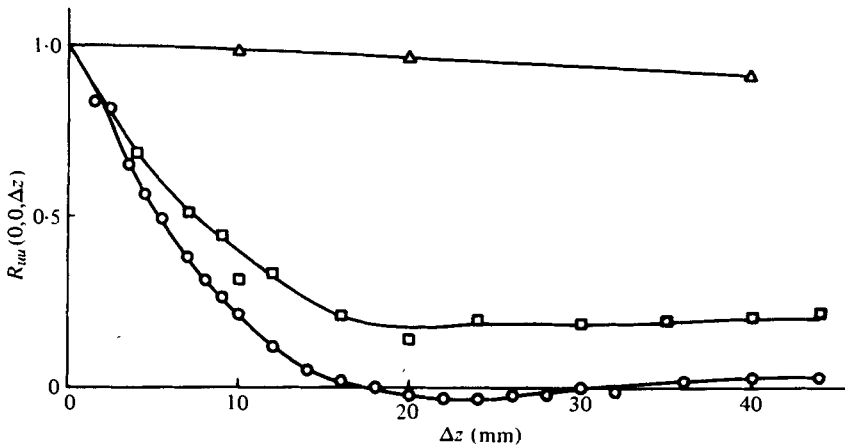


FIGURE 3. The effect of turbulence on the $R_{uu}(0, 0, \Delta z, 0)$ correlation; $U_1/U_2 = 0.4$ and $x = 500$ mm, case (i) (b). \circ , $\eta(\bar{U} = U_1 + 0.5\Delta U)$; \square , $\eta(\bar{U} = U_1 + 0.1\Delta U)$; Δ , $\eta(\bar{U} \rightarrow U_1)$.

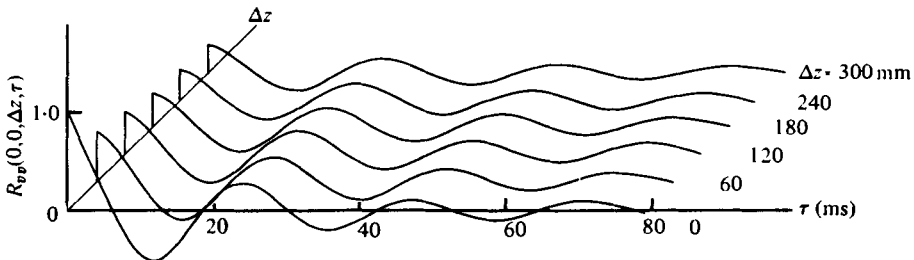


FIGURE 4. Cross-correlation of the v signal; case (i) (b) low-pass filtered at 50 Hz, $U_1/U_2 = 0.4$, $x = 1100$ mm, $U = U_1 + 0.7(U_2 - U_1)$.

scales resulting from similar measurements were interpreted by Chandrsuda *et al.* (1978), as evidence for the non-existence of large coherent eddies.

The interpretation of these results alone is, however, quite difficult. One may attribute the decrease of R_{uu} with increasing turbulence level to a supposition that the large eddies are not as intense as the smaller-scale 'energy-containing' eddies (Townsend 1956). The latter, being three-dimensional and occurring randomly, cause a reduction in the lateral integral scales deduced from the R_{uu} correlation. Thus, R_{uu} measurements in the fully turbulent region will tend to mask an important feature of the large coherent eddies. A second interpretation, no less plausible, stems from the fact that an array of cylindrical eddies aligned with the x axis induce, as they pass by a stationary observer, velocity fluctuations normal to the radius vector emanating from the centre of each vortex. At the edges of the mixing layer the induced fluctuations are predominantly in the direction of streaming, u , but in the central region the fluctuations induced by the large eddies are in the transverse, v , direction. On this premise the measurement of $R_{vv}(0, 0, \Delta z, \tau)$ correlation was undertaken. The R_{vv} correlations shown in figure 4 were taken directly downstream of the splitter plate at $x = 1100$ mm where $U \approx U_1 + 0.7(U_2 - U_1)$ and were filtered at 50 Hz to remove the effect of small-scale fluctuations at small Δz . The R_{vv} correlation remained fairly large for separation distances comparable to the width of the test section, reinforcing

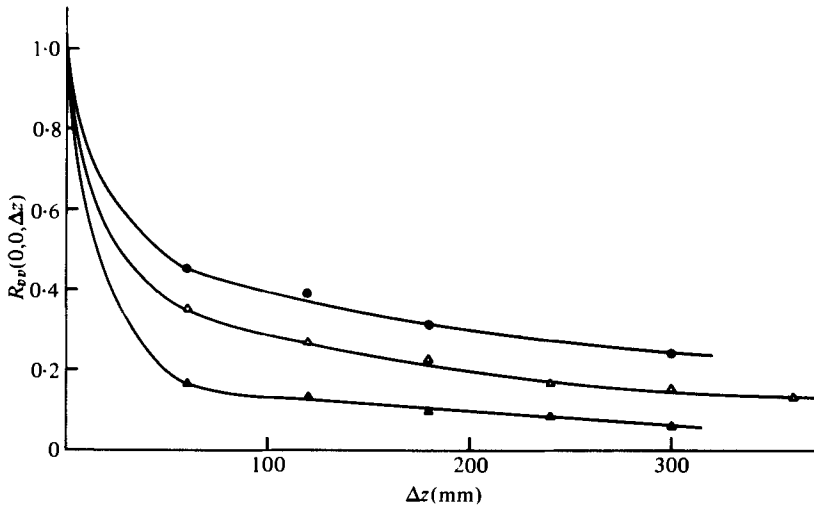


FIGURE 5. $R_{vv}(0, 0, \Delta z, 0)$ for the tripped and non-tripped case $U_1/U_2 = 0.4$; $x = 1100$ mm, $U = U_1 + 0.7(U_2 - U_1)$. ●, case (i) (b), low-pass filtered 50 Hz; Δ, no trip, low-pass filtered 50 Hz; ▲, no trip, wide band.

the notion that a periodic structure which is coherent in transverse z co-ordinate does indeed exist. The unfiltered R_{vv} correlations measured at the same Δz intervals were quantitatively the same with the exception of the autocorrelation $R_{vv}(0, 0, 0, \tau)$. For $\tau = 0$, the unfiltered correlation was approximately double the $R_{vv}(0, 0, 0, 0)$ filtered at 50 Hz. In figure 5 the $R_{vv}(0, 0, \Delta z, 0)$ correlations for the tripped and untripped shear layer are compared. It is found that the degree of coherence is less pronounced, though present in the untripped case.

With the largest probe separation of $\Delta z = 300$ mm cross-correlation of the products $u_1 u_2$ and $v_1 v_2$ were taken at various positions across the flow. These products represent non-normalized correlation measurements. While the $v_1 v_2$ correlations retained their coherence in the central region of the flow (figure 6a), the $u_1 u_2$ correlations became insignificant (figure 6b). Outside the turbulent zone both correlation measurements indicate periodicity and spanwise coherence of the large structures, but only $v_1 v_2$ measurements bring out the two-dimensional feature of the large eddies in the highly turbulent zone.

The spanwise coherence of the large structures was also observed in the heated, single-stream, mixing layer by Fiedler, Korschelt & Mensing (1978). An example of the spatial correlations of temperature fluctuations $R_{TT}(0, 0, \Delta z, 0)$ as measured in this investigation is reproduced in figure 7. In this case the fluctuations correlated are present in the turbulent zone only.

In all measurements described, the thickness of the mixing layer was small in comparison with the height of the test section; thus we were not limited by the proximity of a solid surface. This aspect was further checked by measuring the $R_{uu}(0, 0, \Delta z, 0)$ correlation at increasing distance from the edge of the mixing layer. The correlation started to decrease very slowly as 'potential' fluctuations emanating from the wall boundary layer were felt.

Although the correlation coefficients $R_{vv}(0, 0, \Delta z, \tau)$ and $R_{TT}(0, 0, \Delta z, \tau)$ are repetitive

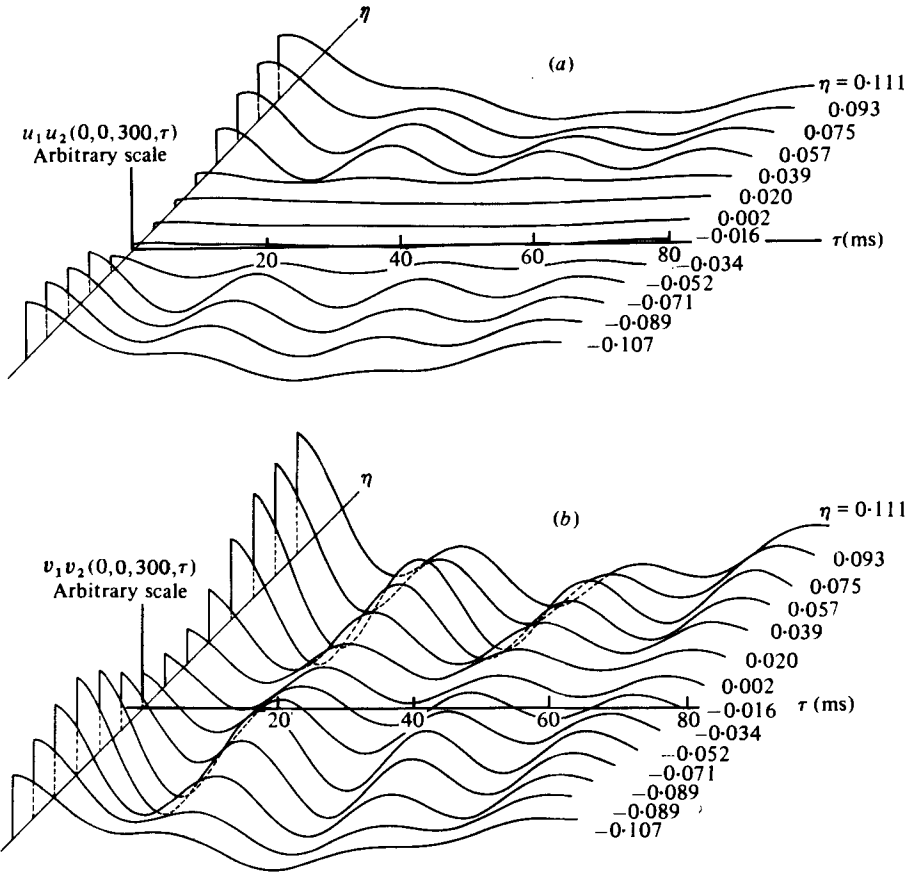


FIGURE 6. Cross-correlations of unfiltered u and v signals; case (i) (b); $U_1/U_2 = 0.4$; $x = 1100$ mm, $\Delta z = 300$ mm. (a) u_1u_2 , (b) v_1v_2 .

and finite at large separation distances across the entire flow and $R_{uu}(0, 0, \Delta z, \tau)$ is finite at large Δz outside of the turbulent region, the degree of two-dimensionality of the large eddy-structure can only be inferred from these data. A physical picture of these eddies can be obtained by tagging the fluid and visualizing it. Smoke, introduced from nozzles located near the splitter plate, was used in Tel-Aviv for $U_1/U_2 = 0.4$, while heating was used in Berlin for $U_1/U_2 = 0$. A single smoke filament introduced into a tripped mixing layer is shown in figure 8(a). The smoke is concentrated in discrete lumps, which are connected by a thin filament oblique to the direction of streaming. The large eddies are repeatable, but superimposed on them are smaller lumps of smoke, giving the large eddies a rough, corrugated appearance. Almost the entire length of the mixing layer is seen in this picture. Two-dimensional forcing of the mixing layer improves the coherence and repeatability of the large eddy-structure.

In figure 8(b) a time record of temperature distribution across the mixing layer is shown. The record was obtained from an array of nine sensors located at $x = 400$ mm and separated in y by 20 mm gaps. The abscissa in this figure is time, while the ordinate represents temperature. Each trace is displaced vertically according to its

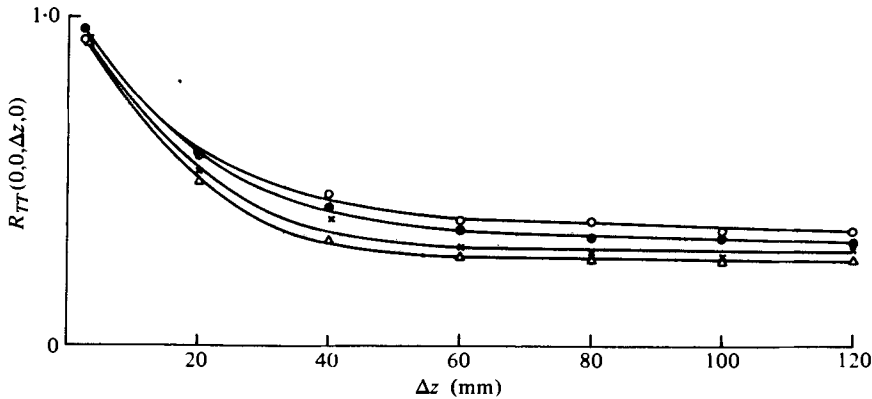


FIGURE 7. Temperature correlations in a mixing layer downstream of the splitter plate $U_1/U_2 = 0$, $y = 0$ (courtesy of Fiedler *et al.* 1978). x values (mm): ○, 700; ●, 600; ×, 500; △, 400.

position in the array, thus the figure represents a passage of a single eddy in the y, t plane.

The lowest trace represents a sensor which is mostly located in the warm streaming flow, while the uppermost trace corresponds to a sensor located at the outer edge of the mixing layer, and exposed most of the time to the colder ambient fluid. The forced mixing layer consists of very repeatable, lumpy vortices, which are connected by a thin vortex sheet. The two streams in their unmixed state are separated by the thin sheet, as may be seen from the temperature jump across it. The engulfment of the hot fluid on one side and cold fluid on the other side of the sheet into the coiled, lumpy vortex can be deduced from this figure. Most of the turbulent activity occurs at the outer rim of the vortex. Although the inner core of the vortex is inactive, the fluid in it appears to be well mixed and its temperature is an intermediate temperature of the two streams. An array of lumpy vortices connected by thin sheets oblique to the direction of streaming is also shown in figure 8(c) for $U_1/U_2 = 0.4$. In this case, smoke was used for visualization of a forced mixing layer at a frequency of 40 Hz. Consequently, the large coherent eddies exist in a single stream mixing layer as well as in the mixing layer between parallel streams.

The two-dimensional character of the eddies is shown in figure 9 (plate 2). An array of sensors located at $x = 500$ mm; $y = -30$ mm and separated by 20 mm gaps (i.e. total separation was 160 mm) gave the temperature record shown in figure 9(a). The mixing layer was heated, $U_1/U_2 = 0$, but it was not forced. Temperature fluctuations appeared at fairly regular intervals and occurred simultaneously at most z positions. The duration of these fluctuations vary somewhat from sensor to sensor and from event to event, suggesting that the vortices are not perfectly cylindrical. The eddies are contorted, corrugated and sometimes slightly skewed, but they generally span across the entire flow. These characteristics are reflected in the correlation coefficient, which is always smaller than unity.

Two-dimensional forcing improves the spanwise regularity of the large eddies (figure 9b), but does not remove the corrugations, which are caused by the smaller scales over which no control is exercised.

The spanwise visualization of the eddies by smoke is much more difficult and it

could only be provided in the forced case. The smoke was emitted from five holes in a 1.2 mm tube, which was stretched across the test section. A sheet of light approximately 1–2 mm thick, provided by a 5 W laser, illuminated the flow in a plane corresponding to $U = U_1 + 0.5(U_2 - U_1)$ (figure 9c). The two-dimensional character of the larger structures could thus be observed without ambiguity. Unfortunately the light was too weak to provide the reader with photographs showing individual realizations. The flow was thus forced at 60 Hz and the laser beam was chopped at the same frequency; causing the pattern to be repeatable in time and enabling us to use long exposure times (2 s). Consequently, the photograph shown (figure 9c) represents an ensemble of 120 realizations. The small-scale structures are smeared by the averaging process, but the large coherent eddies exhibit clearly their two-dimensional character. The wide white strip in the figure marks the centre-line of the test section (continuation of the splitter plate) with tick marks every 250 mm. Since this strip is 10 mm wide, the thickness of the sheet of light and its inclination to the splitter plate can be estimated by comparison. The sheet of light is seen in the figure as a thin line, intersecting the centre-line strip at a small angle. It should be noted that all photographs were taken at the same velocities as the measurements.

We finally addressed ourselves to the effects of the free-stream turbulence on the two-dimensional character of the coherent eddy structure by placing a coarse grid 250 mm upstream of the trailing edge. The turbulence in the free stream was increased by a factor of 15 (it was above 3% at the initiation of mixing) but the R_{uu} correlation outside the mixing layer did not diminish at large distances between the measuring probes. The shape of the correlation curve changed appreciably [see figure 2, case (iv)] because the decaying grid turbulence contributes significantly to the overall correlation coefficient at small separation distances. It should be remembered that the characteristic length of the grid is 17.5 mm and, for distances exceeding this length, the R_{uu} correlation changes very little (figure 2). The value of R_{uu} at large separation distances in case (iv) is 0.4, because the intensities $[\overline{(u_1)^2}]^{\frac{1}{2}}$ and $[\overline{(u_2)^2}]^{\frac{1}{2}}$ used for normalizing R_{uu} have almost doubled. Thus case (iv) can be regarded as an indication, that the large coherent eddies contain little energy, otherwise one could not explain how an increase in the free-stream turbulence level could cause such a reduction in R_{uu} outside the mixing layer at small separation distances, yet have no influence on $\partial R_{uu}/\partial z$ at large separation distances (Δz). We think that the disparity between the scales of externally imposed turbulence and the large eddies in the mixing layer actually accentuates the two-dimensional character of the coherent structures.

It is thus concluded that the two-dimensional character of the coherent eddies perseveres in spite of strong external small-scale buffeting, and the 'Brown-Roshko' structure may be more common in practice than was hitherto observed. These conclusions are in direct contradiction with the findings of Chandrsuda *et al.* (1978) although the quantitative results derived from measurements agree fairly well.

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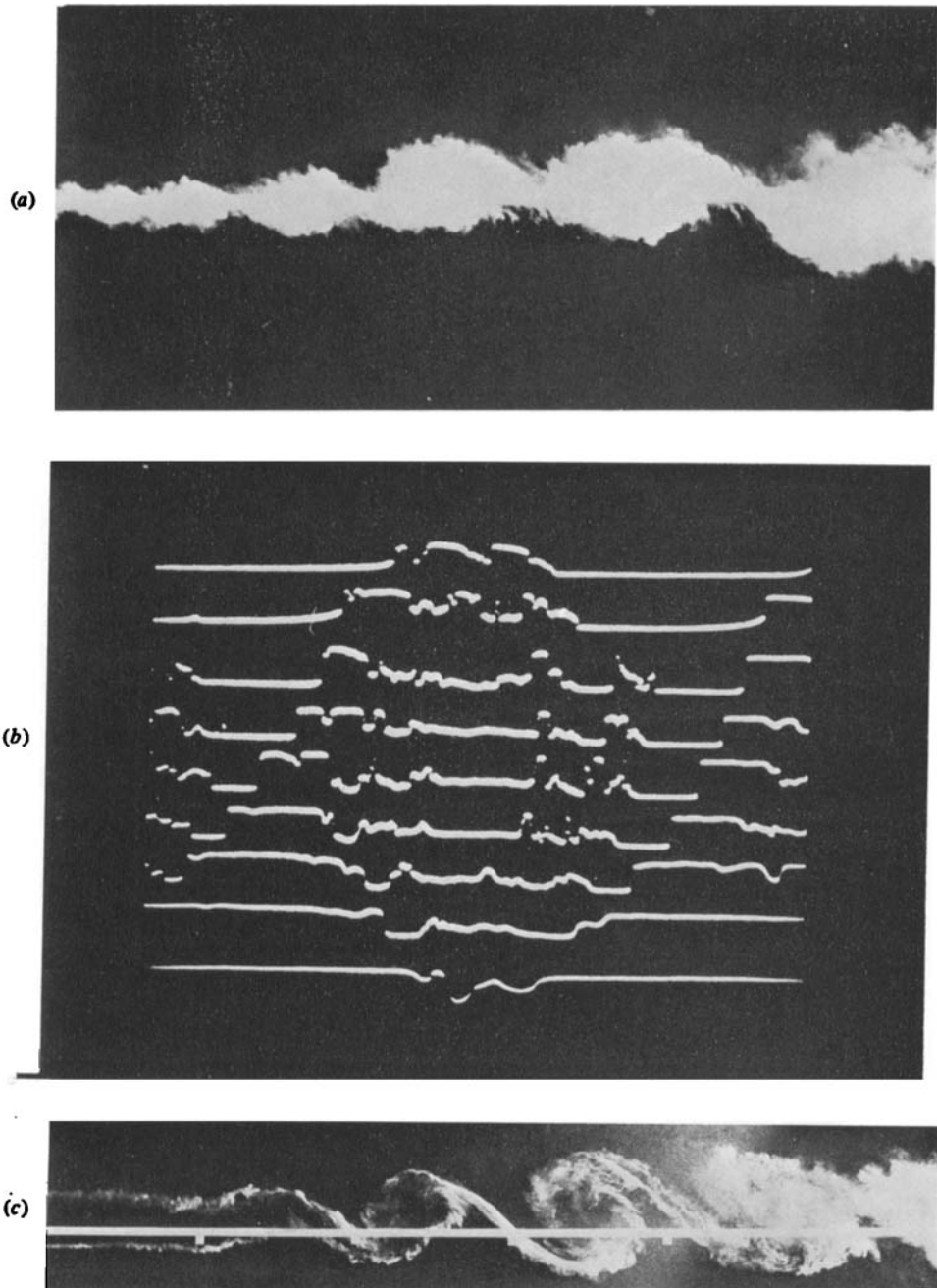


FIGURE 8. Visualization of large vortical structures by smoke and temperature tagging (elevation view). (a) Tripped but not forced mixing layer $U_1/U_2 = 0.4$. (b) Forced case $f = 15$ Hz, $U_1/U_2 = 0$; the abscissa represents $t = 100$ ms. (c) Forced case $f = 40$ Hz, $U_1/U_2 = 0.4$, the distance between white tick marks = 250 mm.

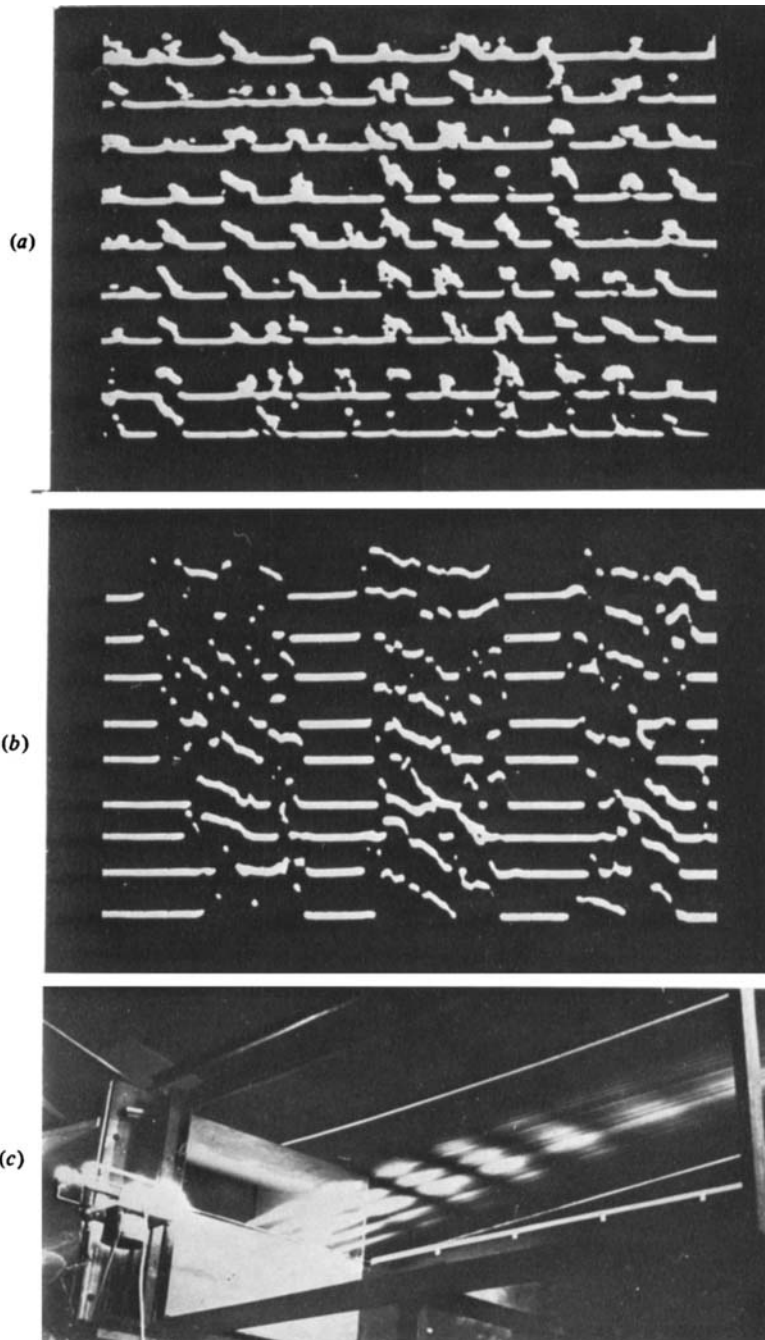


FIGURE 9. Spanwise distribution of temperature and smoke filaments (plan view). (a) Unforced case $U_1/U_2 = 0$, $t = 500$ ms. (b) Forced case $f = 15$ Hz, $U_1/U_2 = 0$, $t = 200$ ms. (c) Forced case $f = 40$ Hz, $U_1/U_2 = 0.4$, distance between white tick marks = 250 mm.